A SYNOPSIS

OF

ELEMENTARY RESULTS

IN

PURE AND APPLIED MATHEMATICS:

CONTAINING

PROPOSITIONS, FORMULÆ, AND METHODS OF ANALYSIS,

WITH

ABRIDGED DEMONSTRATIONS.

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INDICES.

29 Multiplication: $a^{\frac{1}{3}} \times a^{\frac{1}{2}} = a^{\frac{1}{3}+\frac{1}{2}} = a^{\frac{5}{6}}$, or $\sqrt[6]{a^5}$;

$$a^{\frac{1}{m}} \times a^{\frac{1}{n}} = a^{\frac{1}{m}+\frac{1}{n}} = a^{\frac{m+n}{mn}}, \text{ or } \sqrt[mn]{a^{m+n}}.$$

Division: $a^{\frac{2}{3}} \div a^{\frac{1}{2}} = a^{\frac{2}{3}-\frac{1}{2}} = a^{\frac{1}{6}}$, or $\sqrt[6]{a}$;

$$a^{\frac{1}{n}} \div a^{\frac{1}{m}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}}, \text{ or } \sqrt[mn]{a^{m-n}}.$$

Involution: $(a^{\frac{2}{3}})^{\frac{1}{2}} = a^{\frac{2}{3}\times\frac{1}{2}} = a^{\frac{1}{3}}$, or $\sqrt[3]{a}$.

Evolution: $\sqrt[7]{a^{\frac{2}{3}}} = a^{\frac{2}{3}\times\frac{1}{7}} = a^{\frac{2}{21}}$, or $\sqrt[21]{a^2}$.

$$a^{-n} = \frac{1}{a^n}, \quad a^0 = 1.$$

HIGHEST COMMON FACTOR.

30 Rule.—To find the highest common factor of two expressions—*Divide the one which is of the highest dimension by the other, rejecting first any factor of either expression which is not also a factor of the other. Operate in the same manner upon the remainder and the divisor, and continue the process until there is no remainder. The last divisor will be the highest common factor required.*

31 Example.—To find the H. C. F. of

$3x^5-10x^3+15x+8$ and $x^5-2x^4-6x^3+4x^2+13x+6$.

	1– 2– 6+ 4+13+ 6	3+0–10+ 0+15+ 8	3
	3	–3+6+18–12–39–18	
1	3– 6–18+12+39+18	2) 6+ 8–12–24–10	
	–3– 4+ 6+12+ 5	3+ 4– 6–12– 5	
	2) –10–12+24+44+18	–3– 9– 9– 3	
	– 5– 6+12+22+ 9	– 5–15–15– 5	
	3	+ 5+15+15+ 5	
5	–15–18+36+66+27		
	+15+20–30–60–25		
	2) 2+ 6+ 6+ 2	Result H. C. F. $= x^3+3x^2+3x+1$.	
	1+ 3+ 3+ 1		

32 Otherwise.—To form the H. C. F. of two or more algebraical expressions—*Separate the expressions into their simplest factors. The H. C. F. will be the product of the factors common to all the expressions, taken in the lowest powers that occur.*

LOWEST COMMON MULTIPLE.

33 *The L. C. M. of two quantities is equal to their product divided by the H. C. F.*

34 Otherwise.—To form the L. C. M. of two or more algebraical expressions—*Separate them into their simplest factors. The L. C. M. will be the product of all the factors that occur, taken in the highest powers that occur.*

Example.—The H. C. F. of $a^2(b-x)^5c^7d$ and $a^3(b-x)^2c^4e$ is $a^2(b-x)^2c^4$; and the L. C. M. is $a^3(b-x)^5c^7de$.

EVOLUTION.

To extract the square root of

$$a^2-\frac{3a\sqrt{a}}{2}-\frac{3\sqrt{a}}{2}+\frac{41a}{16}+1.$$

Arranging according to powers of a, and reducing to one denominator, the expression becomes $\dfrac{16a^2-24a^{\frac{3}{2}}+41a-24a^{\frac{1}{2}}+16}{16}$.

35 Detaching the coefficients, the work is as follows:—

$$\begin{array}{l|l}
 & 16-24+41-24+16\ (4-3+4 \\
 & 16 \\
\hline
8-3 & -24+41 \\
\quad -3 & 24-9 \\
\hline
8-6+4 & 32-24+16 \\
 & -32+24-16 \\
\hline
\end{array}$$

Result $\dfrac{4a-3a^{\frac{1}{2}}+4}{4}=a-\frac{3}{4}\sqrt{a}+1.$

356 No expression with integral coefficients such as $A+Bx+Cx^2+\ldots$ can represent primes only.

PROOF.—For it is divisible by x if $A=0$; and if not, it is divisible by A, when $x=A$.

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357 The number of primes is infinite.

PROOF.—Suppose p the greatest prime. Then the product of all primes up to p plus unity is either a prime, or divisible by a prime greater than p.

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358 If a be prime to b, and the quantities $a, 2a, 3a, \ldots (b-1)a$ be divided by b, the remainders will be different.

PROOF.—Assume $ma-nb=m'a-n'b$, m and n being less than b, $\therefore \quad \frac{a}{b}=\frac{n-n'}{m-m'}$ Then by (350).

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359 A number can be resolved into prime factors in one way only. By (353).

360 To resolve 5040 into its prime factors.

RULE.—*Divide by the prime numbers successively.*

$$\begin{array}{r|l} 2\times 5 & 5040 \\ 2 & 504 \\ 2 & 252 \\ 2 & 126 \\ 7 & 63 \\ 3 & 9 \\ & 3 \end{array}$$

Thus $5040 = 2^4.3^2.5.7$.

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361 Required the least multiplier of 4704 which will make the product a perfect fourth power.

By (196), $4704 = 2^5.3.7^2$.

Then $2^5.3^1.7^2 \times 2^3.3^3.7^2 = 2^8.3^4.7^4 = 84^4$.

The indices 8, 4, 4 being the least multiples of 4 which are not less than 5, 1, 2 respectively.

Thus $2^3.3^3.7^2 = 3528$ is the multiplier required.

362 All numbers are of one of the forms $2n$ or $2n+1$
" " $2n$ or $2n-1$
" " $3n$ or $3n\pm1$
" " $4n$ or $4n\pm1$ or $4n+2$
" " $4n$ or $4n\pm1$ or $4n-2$
" " $5n$ or $5n\pm1$ or $5n\pm2$

and so on.

363 All square numbers are of the form $5n$ or $5n\pm1$.

Proved by squaring the forms $5n$, $5n\pm1$, $5n\pm2$, which comprehend all numbers whatever.

364 All cube numbers are of the form $7n$ or $7n\pm1$.
And similarly for other powers.

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365 The highest power of a prime p, which is contained in the product $\lfloor m$, is the sum of the integral parts of

$$\frac{m}{p},\ \frac{m}{p^2},\ \frac{m}{p^3},\ \&c.$$

For there are $\frac{m}{p}$ factors in $\lfloor m$ which p will divide; $\frac{m}{p^2}$ which it will divide a second time; and so on. The successive divisions are equivalent to dividing by

$$p^{\frac{m}{p}}\cdot p^{\frac{m}{p^2}}\ldots\ \&c. = p^{\frac{m}{p}+\frac{m}{p^2}+}$$

EXAMPLE.—The highest power of 3 which will divide $\lfloor 29$. Here the factors 3, 6, 9, 12, 15, 18, 21, 24, 27 can be divided by 3. Their number is $\frac{29}{3} = 9$ (the integral part).

The factors 9, 18, 29 can be divided a second time. Their number is $\frac{29}{3^2} = 3$ (the integral part).

One factor, 27, is divisible a third time. $\frac{29}{3^3} = 1$ (integral part).

$9+3+1 = 13$; that is, 3^{13} is the highest power of 3 which will divide $\lfloor 29$.

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366 The product of any r consecutive integers is divisible by $\lfloor r$.

PROOF: $\dfrac{n(n-1)\ldots(n-r+1)}{\lfloor r}$ is necessarily an integer, by (96).