Other Logics:
What Nonclassical Reasoning Is All About

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Classical logic

What is logic?

A set of techniques for representing, transforming, and using information.

What is classical logic?

A particular kind of logic that has been well understood since ancient times. (Details to follow…)

Classical logic

I should warn you that
nonclassical logic is *not as weird as you may think*.

I'm not going to introduce
"new ways of thinking" that lead
to bizarre beliefs.

What I want to do is make explicit
some nonclassical ways of reasoning
that people have always found useful.

I will be presenting well-accepted research results,
not anything novel or controversial.

Classical logic

300s B.C.:

ARISTOTLE and other Greek philosophers discover that

***some methods of reasoning are truth-preserving*.**

**That is, *if the premises are true,*
the conclusion is guaranteed true,
regardless of what the premises are.**

Classical logic

Example:

All hedgehogs are spiny.
Matilda is a hedgehog.

∴ **Matilda is spiny.**

You do not have to know the meanings of "hedgehog" or "spiny" or know anything about Matilda in order to know that this is a <u>valid</u> argument.

Classical logic

VALID means TRUTH-PRESERVING.

Logic cannot tell us whether the premises are true.

The most that logic can do is
tell us that IF the premises are true,
THEN the conclusions must also be true.

Classical logic

1854:
George Boole points out that
inferences can be represented as formulas **and**
there is an infinite number of valid inference schemas.

$(\forall x)\ hedgehog(x) \supset spiny(x)$
$hedgehog(Matilda)$

$\therefore\ spiny(Matilda)$

Proving theorems (i.e., proving inferences valid)
is done by manipulating formulas.

Classical logic

1931:

Kurt Gödel proves that

classical logic is incomplete

or more precisely that

in any version of classical logic that is powerful enough to include arithmetic, there are inferences that are valid but cannot be proved so.

Classical logic

Many nonspecialists see Gödel's incompleteness proof as a frightening demonstration of human fallibility.

I see it as a technicality.

Classical logic is "incomplete" in a *technical* sense that has to do with methods of proving theorems.

This does not mean that classical reasoning is invalid.

Classical logic

There are much more compelling reasons to go beyond classical logic.

Classical logic

What's *missing* from classical logic:

* **Any consideration of situations other than the actual one.**
(In CL, everything is true or false; there's no way to consider what would be true *if* some other thing *were* true.)

* **Any way to *get more premises*.**
(You can only work with what you have.)

* **Any way to use *uncertain* or *incomplete* information.**
(CL assumes you know everything relevant, and your knowledge can't possibly change.)

Classical logic

Classical logic simply

has nothing to say

in many situations where

for practical purposes,
we need to conclude something,
even if it's fallible.

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Logics of possible worlds

Some interesting technical extensions of CL:

Modal logic **deals with**
what is *possible* or *impossible.*

Deontic logic **deals with**
obligation and permission.

Logics of possible worlds

Modal and deontic logic are closely related to the logic of *quantification* ("all," "some") in CL.

Two familiar theorems of CL:

All X	≡	**not some not-X**
Some X	≡	**not all not-X**

Remember these…

Logics of possible worlds

MODAL LOGIC labels statements as
"possible" and "impossible,"
"necessary" and "not necessary,"
as well as true or false.

Some axioms (not the whole set):

If necessary-X then X.
If not-possible-X then not-X.

Necessary-X	≡	not-possible not-X
Possible-X	≡	not-necessary not-X

Logics of possible worlds

Look at those last two axioms again…

Necessary-X	≡	not-possible not-X
Possible-X	≡	not-necessary not-X

Compare to two theorems from classical logic:

All X	≡	not some not-X
Some X	≡	not all not-X

Idea: **“Necessary” and “possible” can be understood as “in all/some possible worlds.”**

Logics of possible worlds

DEONTIC LOGIC labels statements as "permitted" and "not permitted," "obligatory" and "not obligatory," as well as true or false.

Some axioms (*these will look familiar*):

Obligatory-X	$\equiv$	**not-permissible not-X**
Permissible-X	$\equiv$	**not-obligatory not-X**

"Obligatory" can be understood as "in all permissible worlds."

Logics of possible worlds

Modal logic is needed to
***reason about hypothetical situations*.**

Deontic logic is needed to
***reason about duties*.**

Both involve interesting (and unsolved)
technical problems:

Exactly what axioms should we add
to classical logic
to get things to come out right?

Logics of possible worlds

Additional technical problems in deontic logic:

- Apparent obligations
  (Can you ever be so *sure* of your duty that no possible additional knowledge could change it?)

- Contrary-to-duty obligations
  (What if you've done something impermissible?)

Without contradiction, we want to be able to say, "Don't do X, but if you do X, do Y" (e.g., pay reparations).

Logics of possible worlds

A practical example: Asimov's laws of robotics (1940).

(1) A robot may not injure a human being, or, through inaction, allow a human being to come to harm.

(2) A robot must obey orders given it by human beings, except where such orders would conflict with the First Law.

(3) A robot must protect its own existence as long as such protection does not conflict with the First or Second Law.

Note the crucial roles of:

- deontic logic (duties)
- modal logic (hypothetical situations)
- priority ranking (defeasible logic, which we'll get to).

Contents

Induction

The sun rose today.
The sun rose yesterday.
The sun rose the day before.
And so on…

∴ **The sun will rise tomorrow.**

Is this a valid inference?
It is certainly nonclassical!

Induction

Induction is the only kind of logic
that enables you to
get new knowledge,
not just manipulate and unpack
the knowledge you already have.

But what *is* induction, and should we trust it?

Induction

This is a vexing problem in the philosophy of science.

There is no *logical* reason why a long series of previous sunrises *should* imply a future sunrise.

And our level of certainty varies.

We trust induction more
if we have made the observations repeatedly
under a wide variety of conditions.

Induction

Well-kept secret (ask any philosopher):

There is no single, fixed "Scientific Method" for distilling Data into Truth.

Instead, we have varying levels of confidence depending on how well we think we've pinned down the conditions under which something happens.

Techniques:

- **Controlled experiments**
- **Replicability**
- **Statistical tests**

Induction

Sir Karl Popper:

There is actually no “inductive logic” at all.

Instead, we have *hypotheses that have survived tests*.

The hypothesis “The sun rises every day”
has been tested so many times, under different conditions, that we have confidence in it.

Induction

I think Popper is basically right, *but*…

- Hypotheses have to be *vulnerable* (as he points out). That is, it has to be *possible* to test a hypothesis.

  (Beware of “Jeane Dixon theories” that are “true” no matter what happens.)

- Something has to *lead us to propose the hypothesis* in the first place, and to think that the hypothesis is interesting and useful.

Contents

Logics of uncertainty

In real life, we cannot classify all our premises neatly as “true” or “false” because:

- Some knowledge is genuinely uncertain.

- Some statements are true only to a degree (e.g., “Covington is bald.”)

  Would I be bald if I had only 1 hair?
  Only 2 hairs?
  Only 3 hairs?
  …
  Only 1500 hairs?

Logics of uncertainty

Bayesian inference **uses probability theory to make probabilistic inferences.**

Bayes' Theorem **(Rev. Thomas Bayes, 1764):**

$$P(B|A) = [P(A|B) \times P(B)] \, / \, P(A)$$

Example:

$P(B\|A) = ?$	**Prob. that patient has meningitis, given stiff neck**
$P(A) = 0.10$	**10% of the patients have stiff necks**
$P(B) = 0.01$	**1% of the patients have meningitis**
$P(A\|B) = 0.5$	**50% of those with meningitis have stiff necks**

We find $P(B|A) = [0.5 \times 0.01] \, / \, 0.10 \;=\; 0.05 \;=\; 5\%$

Logics of uncertainty

Putting it more simply,

Bayes' Theorem deals with the difference between

"Most fire trucks are red"

and

"Most red things are fire trucks."

Logics of uncertainty

Fuzzy logic (Lotfi Zadeh, 1960s)
deals with conditions that are *true to a degree*.

P(*statement*) ranges from 0 to 1.

Here is one of several systems of logical operators:

$P(\text{not } X) = 1 - P(X)$
$P(X \text{ and } Y) = \min(P(X),P(Y))$
$P(X \text{ or } Y) = \max(P(X),P(Y))$

FL is popular with engineers
as a way of mixing logic with arithmetic.
It does not solve any deep philosophical problems.

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Logic for dealing with partial knowledge

Much human reasoning is *nonmonotonic*.

That is:

we reach conclusions *tentatively*
which we will abandon
if given further information.

The reason?

We are accustomed to working with
partial knowledge.

Logic for dealing with partial knowledge

Example:

I have a bird named Tweety.

(Do you think Tweety can fly? Your best guess?)

Now suppose I tell you Tweety is an ostrich.

(Do you still think Tweety can fly?)

Logic for dealing with partial knowledge

What's going on?

Human knowledge is naturally organized into GENERAL CASES and EXCEPTIONS.

This can involve many layers:
general rule, exception, exception to exception, etc.

Each of Asimov's laws of robotics is an exception to the preceding laws.

Logic for dealing with partial knowledge

There are many systems of default or defeasible reasoning, but in what follows, I'll be giving you that of Donald Nute (University of Georgia).

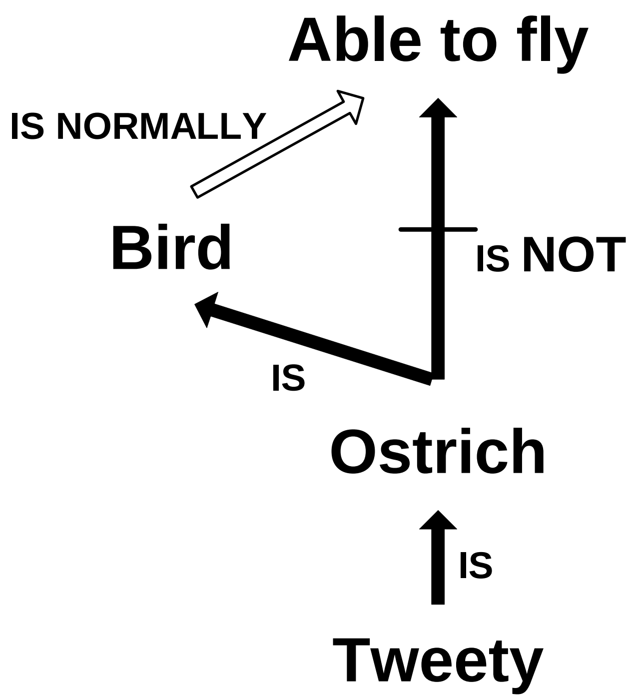
- Rules are ranked in order of precedence.

- Unless specified otherwise, more specific rules have precedence over more general ones.

  (E.g., "ostriches don't fly" has precedence over "birds fly," because ostriches are a subset of birds.)

Logic for dealing with partial knowledge

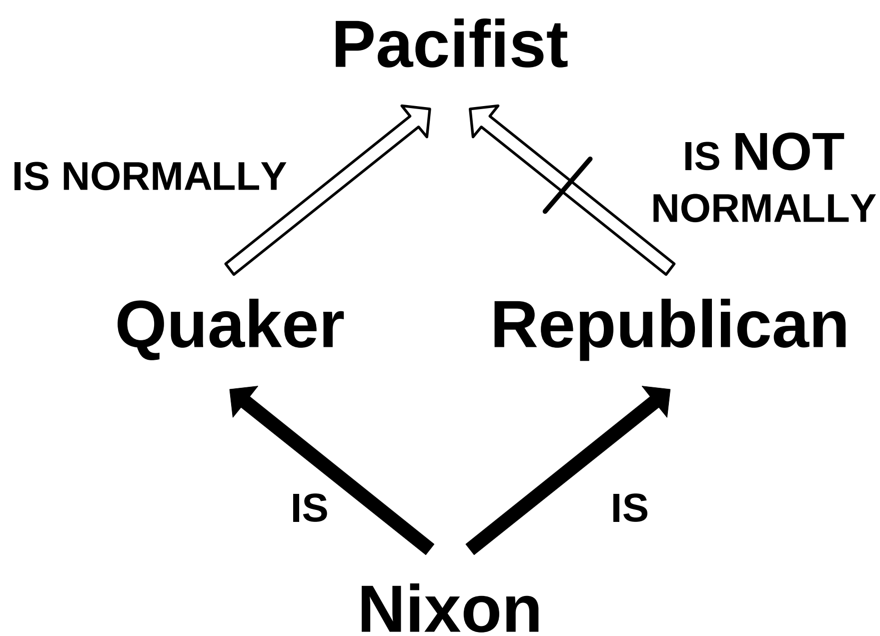
The famous “Tweety triangle” –



Logic for dealing with partial knowledge

Sometimes you can't reach a conclusion.

Example: The "Nixon diamond" –



Logic for dealing with partial knowledge

What good is defeasible logic?

- **Describing knowledge that includes tentative or partial information**

- **Encoding the results of induction (which can be modified by more specific knowledge in the future)**

Logic for dealing with partial knowledge

More applications of defeasible logic

- **Encoding complex conditions in a concise way that is easy for humans to understand (Covington, embedded microcontroller work)**

- **Explaining quirks of the human mind (Hudson, in *Language,* 2000, argues that the reason English has no contraction for *am not* is that 2 rules of grammar get into a Nixon diamond.)**

CONCLUSIONS

- Logic is not a dead subject;
  most of it has yet to be discovered/invented!

- Nonclassical logic is essential
  for practical use of information.

- As computers become *information machines*
  instead of just *arithmetic machines,*
  logic will form an increasingly important basis
  for computer technology.

- Any questions? -